# The Academy of Woodlands

## Maths & Calculation Policy

**Sept 2021**

The National Curriculum 2014 has 3 central aims:

- Become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately. In order to achieve this we need to provide opportunities for children to investigate numbers by counting, cardinality (how many there are in the group), comparison and composition. They need to practice decomposing (breaking numbers into parts) and recomposing numbers, recalling number bonds and multiplication tables to improve mathematical fluency.

- **Reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language. The conversations we have and the questions we ask are key to developing reasoning skills. We can ask children to describe, explain, convince others, justify and prove to promote their reasoning skills. Adults can support children to develop reasoning by modelling, using mathematical language (also displayed in classrooms) and group work.

- Can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions. Activities should be provided where children can solve number problems, practical problems and missing number problems. Problem solving is not just about solving the problem; it is about how they solved the problem. What strategies and mathematical concepts did they use? All pupils should have the opportunity to apply their mathematics to solve problems. The use of mathematical language, modelling and the bar model can all help support children to develop their problem solving skills. Higher attaining children need to solve problems that require more demanding reasoning and problem solving skills rather than harder numbers. We must ensure that children have the opportunity to conjecture when problem solving. Problem solving is more than learning and following a procedure.

**Concrete, pictorial and Abstract (CPA) approach**

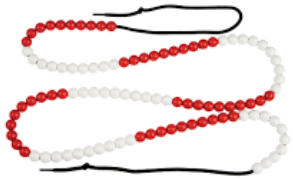
The **Concrete Pictorial Abstract (CPA)** approach is highly effective in the teaching of Maths to develop conceptual understanding. This approach will vary between year groups and the individual abilities of children within each class.

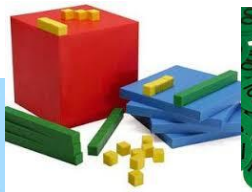
Objects, pictures, words, numbers and symbols are everywhere. The mastery approach incorporates all of these to help children explore and demonstrate mathematical ideas, enrich their learning experience and deepen understanding. Together, these elements help cement knowledge so pupils truly understand what they've learnt.

All pupils, when introduced to a key new concept, should have the opportunity to build competency in this topic by taking this approach. Pupils are encouraged to physically represent mathematical concepts. Objects and pictures are used to demonstrate and visualise abstract ideas, alongside numbers and symbols.

Concrete – The doing stage

There is a clear focus on the use of manipulatives and visual images to support understanding in every year group. Each new concept or calculation strategy will be introduced using appropriate manipulatives, giving the children a clear picture of the theoretical mathematics they are learning. It is important that children have access to a wide range of manipulatives in every year group and, consequently, we encourage children to be independent in their use of manipulatives throughout the school and access resources as they see fit. This is the foundation for conceptual understanding. These resources will vary depending on year group and individual needs.
Concrete resources that may be found in classrooms will include:




Pictorial – The seeing stage

A child has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or a picture of the problem.

Abstract- The symbolic stage

A child is now capable of representing problems by using mathematical notation, for example 10 ÷ 2 = 5

# Foundation

## Addition

**-Say the number that is one more than a number from 1 to 20.**
**-Find the total number of items in two groups by counting all of them.**
**-In practical activities and discussion, beginning to use the vocabulary involved in adding when combining two groups.**
**-Count on and back from a number other than 0.**

**Vocabulary**
Add, more, make, sum, total, altogether, one more, two more, ten more, how many more to make..?, how many more is…than…?

**Resources**
Oral and practical work
Songs and rhymes
Dice and number games
Number stories for combining sets eg 3 pigs in a field, 2 in a sty how many altogether?
Number track


Number bonds for numbers up to 10
Full number lines

## Subtraction

**-Say the number that is one less than a number from 1 to 20.**
**-In practical activities and discussion, beginning to use the vocabulary involved in subtraction when taking away objects groups.**

**Vocabulary**
Take (away), leave, how many are left/left over? How many have gone? One less, two less, ten less, how many fewer is..? difference between, is the same as

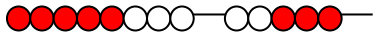
**Resources**
Oral and practical work
Songs and rhymes
Dice and number games, counting back, taking away.
Use of number tracks.


Number stories using objects

Children are encouraged to gain a sense of the number system through the use of counting concrete objects.

They combine objects in practical ways and count all.

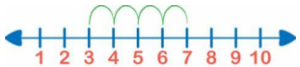

They understand addition as counting on and will count on in ones and twos using objects, cubes, bead string and number line.

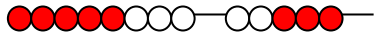


Children are encouraged to gain a sense of the number system through the use of counting concrete objects.

They understand subtraction as counting out.


They begin to count back in ones and twos using objects, cubes, bead string and number line.



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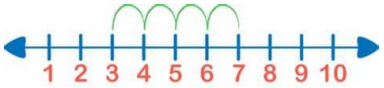
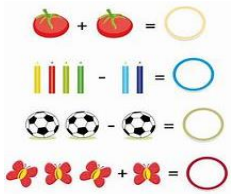


They use concrete and pictorial representation to record their calculations.<br><br>
They begin to use + and =<br>
 <br><br>
They are encouraged to develop a mental picture of the number system in their heads to use for calculations.<br><br>
Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation.
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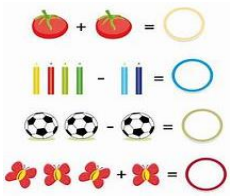
<td>
They use concrete and pictorial representation to record their calculations.<br><br>
They begin to use - and =<br>
 <br><br>
They are encouraged to develop a mental picture of the number system in their heads to use for calculations.<br><br>
Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation.
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Multiplication<br>
-Start to solve problems involving doubling.<br><br>
Vocabulary<br>
Repeated addition, add, double, multiply, how many altogether?<br><br>
Resources<br>
Counting in ones, twos, tens<br>
Odd and even numbers<br>
Matching pairs eg socks<br>
Noah’s ark<br>
Songs and rhymes<br>
Finding doubles in dominoes<br>
Doubles in practical contexts.<br>
Groups of objects with the same number, counting how many in each group, and finding how many altogether
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Division<br>
-Start to solve problems involving halving and sharing<br><br>
Vocabulary<br>
Sharing, equal, halving, divide, how many each?<br><br>
Resources<br>
Practical activities, songs and rhymes<br>
Sharing during snack time by giving 1 each<br>
Is there an easier way of sharing a larger amount? Eg 2 at a time<br>
Making groups/piles of 2, finding partners<br>
Eg in PE grouping in 2s, how many pairs are there?<br>
1 ball for each pair, how many balls do I need to get out?
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<td>
Children use concrete objects to make and count equal groups of objects.<br><br>
They will count on in twos using a bead string and number line.
</td>
<td>
Children use concrete objects to count and share equally into 2 groups.<br><br>
6 cakes shared between 2 people each person gets 3 cakes. 6 ÷2 = 3
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</table>

They understand doubling as repeated addition.

2 + 2 = 4

They use concrete and pictorial representation to record their calculations.

Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation.


They count a set of objects and halve them by making two equal groups.

They understand sharing and halving as dividing by 2.

They will begin to use objects to make groups of 2 from a given amount.

They use concrete and pictorial representation to record their calculations.

Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation.

# ADDITION - KS1 (Years 1&2)

**Year 1**

**-Read, write and interpret mathematical statements involving addition (+) and (=) signs.**
**-Represent and use number bonds within 20**
**-Add and one-digit and two-digit numbers to 20, including zero**
**-Solve one-step problems that involve addition using concrete objects and pictorial representations, and missing number problems such as 4 + □ = 7**

**Resources**

Songs and rhymes
Working with apparatus such as bead strings to 20, cubes, dienes, Numicon:
Use + and = signs and associated vocabulary.
Adding more than 2 numbers
Putting the larger number first 13+3=
13 in your head or on fingers
Counting in 10s from multiples of 10
Number bonds of all numbers to 20

**Year 2**

**-Solve addition problems using concrete objects and pictorial representations, including:**

- **a two-digit number and ones**
- **a two-digit number and tens**
- **two two-digit numbers**
- **adding three one-digit numbers**

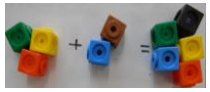
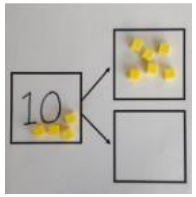
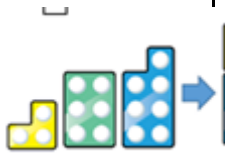
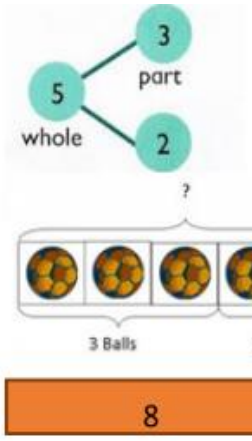
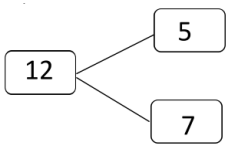
**-Recall and use addition facts to 20 fluently, and derive and use related facts up to 100**
**-Show that addition of two numbers can be done in any order (commutative)**
**-Use the inverse relationship between addition and subtraction to check calculations and solve missing number problems.**

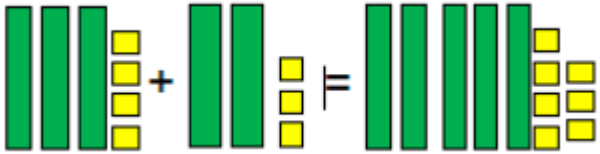
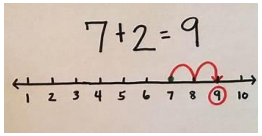
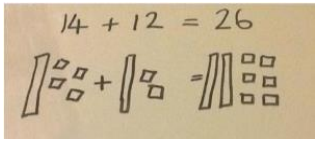
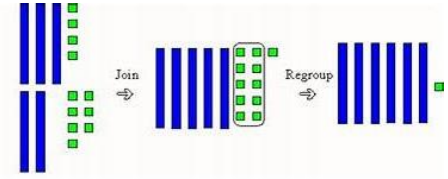
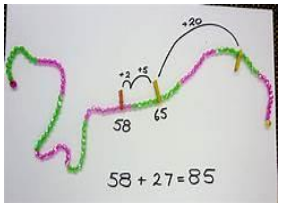
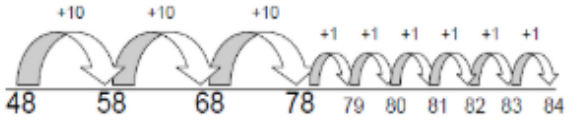
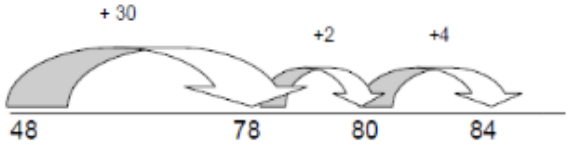
**Resources**

Counting in 10s from any number
Rapid recall of all number bonds for all numbers to 20.
Use of numicon, dienes, bar model, part whole model to demonstrate.

**Vocabulary**

Exchange – I can exchange 10 ones for 1 ten etc

| | Concrete | Pictorial | Abstract |
|---|---|---|---|
| Stage 1 | Use part whole model, numicon, cubes and bead strings to add two numbers together as a group or in a bar.       | Use jottings to represent numbers.   Use pictures to add two num-bers together as a group or in a bar. | Children will record their calculation using a pictorial method along with a calculation using numbers and symbols.   11 + 4 = 15 They may use their fingers to support their mental methods  5 + 2 = 7 |

| | | | |
|---|---|---|---|
| Stage 2 | **Grouping objects to add**<br>Children will use dienes cubes to add larger numbers where regrouping is not required.<br><br>They will also use a bead string to add larger numbers by counting in tens and ones<br>  | **Number line**<br>Start at the larger number on the number line and count on in ones or in one jump to find the answer. Children will show their representations from the concrete method using pictures.<br>  <br>  <br>Numbers will get progressively larger throughout the keystage. Children will be able to add tens and ones using an empty number line. | Children will record their calculation using a pictorial method along with a calculation using numbers and symbols.<br><br>27 + 10 = 37<br><br>27 + 20 = 47<br><br>Children will begin to add multiples of tens.<br><br>27 + □ = 57 |
| Stage 3 | **Partitioning**<br>Children will add larger numbers where they will need to join, exchange and count.<br>  <br>Children will also use bead strings to add numbers together using groups of tens and ones to count on.<br>  | **Number line**<br>Use an empty number line to count in tens and then ones.<br>  <br>When confident:<br>  | **Partitioning**<br>Children will begin to use the partitioning method. Tens and ones will be added to form partial sums and then these partial sums will be added together to find the total.<br><br>25 + 47<br>20 + 5<br>40 + 7<br>20 + 40 = 60<br>5+ 7 =12<br>60 + 12 = 72<br><br>Expanded column when a good understanding of exchanging is demonstrated then formal method.<br><br>24<br>+11<br>5 (4+1)<br>30 (20+10)<br>35 |

# ADDITION - Lower KS2 (Years 3 & 4)

## Year 3

**-Add a range of numbers mentally, including:**
**a three-digit number and ones**
**a three-digit number and tens**
**a three-digit number and hundreds**
**-Add numbers with up to three digits, using formal written methods of columnar addition**
**-Estimate the answer to a calculation and use inverse operations to check answers**
**-Solve problems, including missing number problems, using number facts, place value, and more complex addition.**
**-Add fractions with the same denominator within one whole (for example, $^{5}/_{7} + {}^{1}/_{7} = {}^{6}/_{7}$)**

**Resources**
Dienes
Place value counters
Empty number lines
Part whole model
Bar model

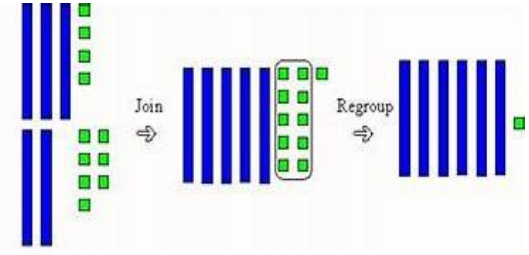
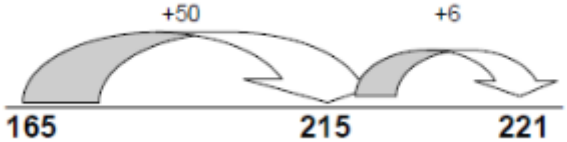
**Vocabulary**
Exchange- I can exchange 11 ones for 1 ten and 1 one

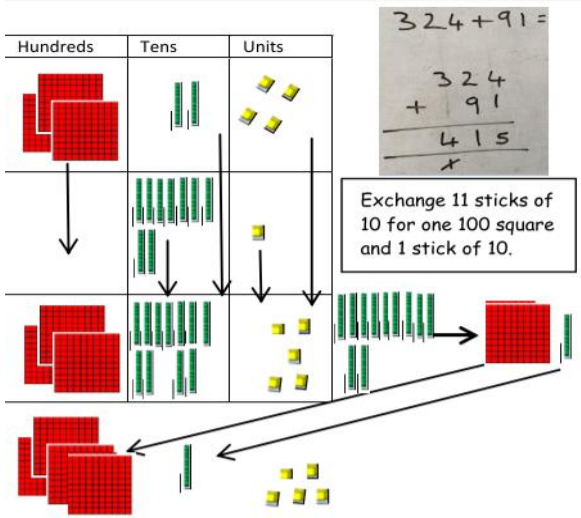
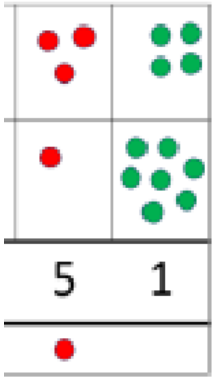
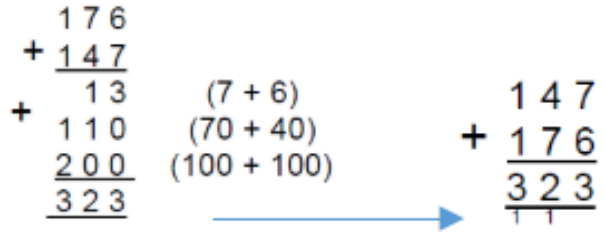
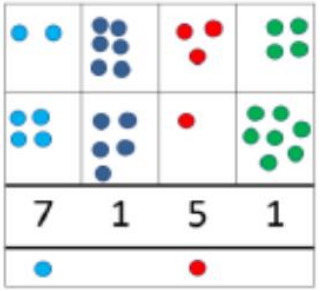
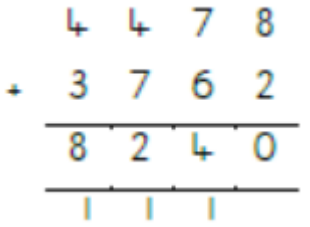
## Year 4

**-Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate**
**-Estimate and use inverse operations to check answers to a calculation**
**-Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.**
**-Add fractions with the same denominator**
**-Solve simple measure and money problems involving fractions and decimals to two decimal places**

**Resources**
Dienes
Place value counters
Empty number lines
Part whole model
Bar model

**Vocabulary**
Exchange- I can exchange 11 hundreds for 1 thousand and 1 hundred

| | Concrete | Pictorial | Abstract |
|---|---|---|---|
| Stage 1 | Use dienes cubes to consolidate learning from KS1. Ensure children are confident at using these to join, exchange and count. This will support them moving onto the next stage of column addition.<br>  | **Number line**<br>Consolidate their learning from KS1 by using an empty number line to count larger numbers.<br>  | **Partitioning**<br>Children will consolidate using the partitioning method. The layout will begin to form a written method to support further progress onto the column method. Hundreds, Tens and ones will be added to form partial sums and then these partial sums will be added together to find the total.<br>200 + 60 + 3<br>+ 100 + 10 + 9<br>300 + 70 + 12<br>300 + 80 + 2 |

| | | | |
|---|---|---|---|
| Stage 2 | Introduce children to place value counters and dienes cubes. Use the column method layout to support their learning onto the abstract method.<br>  | Children can draw a representation of the grid to further support their understanding. Children show the exchange of 10 ones for 1 ten by placing the ten under the tens column.<br>  | **Expanded column method - Formal method**<br>Children to use the Expanded Column Method. Start by partitioning the numbers before the formal column to show the exchange. Once confident, they can move onto the column method in stage 3.<br>  |
| Stage 3 | Children will add larger numbers where they will need to exchange place value counters or dienes cubes.<br>  | Children can draw a representation of the grid using larger numbers. Children show the exchange of ones for tens, tens for hundreds and hundreds for thousands by writing under the correct column. I.e 11 hundreds- exchange for 1 thousand and 1 hundred shown in the written method.<br>  | **Column method - Formal method**<br>Column Method for addition to be used.<br>  <br>Extend to decimals, assigning values to Numicon and bar models to support if needed. |

# ADDITION - Upper KS2 (Years 5 & 6)

## Year 5

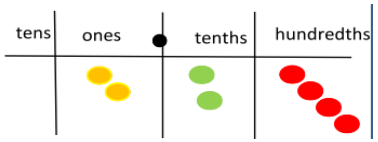
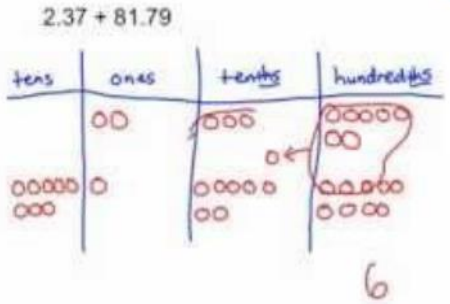
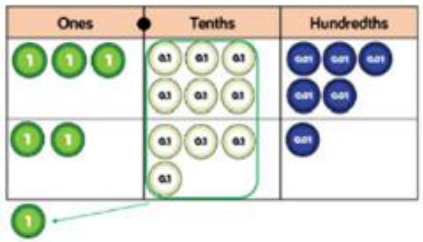
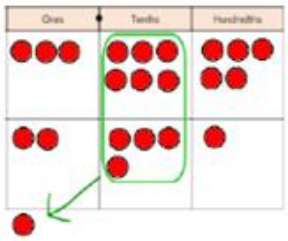
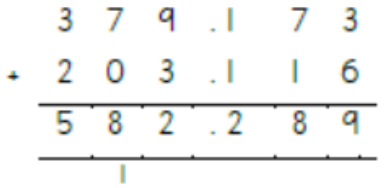
**-Add whole numbers with more than 4 digits, including using formal written methods (columnar addition)**
**-Add numbers mentally with increasingly large numbers (eg. 8 462 + 2300 = 10 762).**
**-Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy**
**-Solve addition and subtraction multi-step problems in contexts, including to 3 decimal places, deciding which operations and methods to use and why.**
**-Add and subtract fractions with the same denominator and denominators that are multiples of the same number**

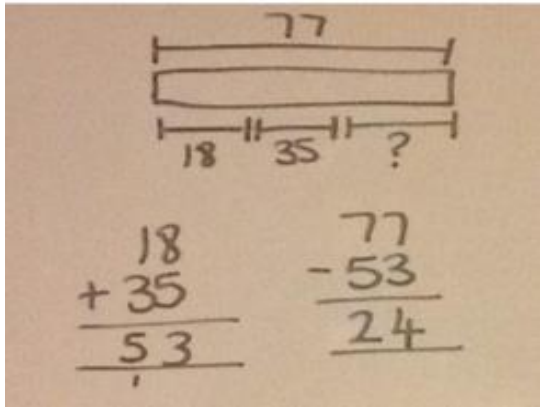
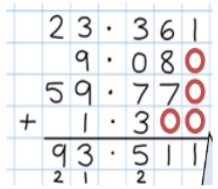
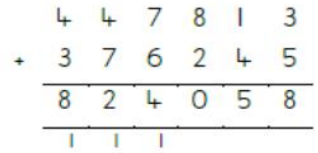
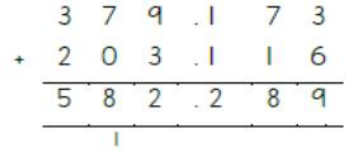
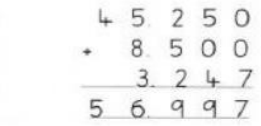
**Resources**
Place value counters
Bar models

## Year 6

**-Add whole numbers with more than 4 digits, including using formal written methods (columnar addition)**
**-Perform mental calculations, including with mixed operations and large numbers**
**-Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why**
**-Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions**

**Resources**
Place value counters
Bar models

| | Concrete | Pictorial | Abstract |
|---|---|---|---|
| Stage 1 | Introduce decimal place value counters and model exchanging for addition.<br>  | Children will draw their representations showing where they have exchanged.<br>  <br>  <br>  | **Column method**<br>Children will continue to develop their understanding of column method addition. Calculations will become larger and include decimal places.<br>  <br>Add fractions with the same denominator and multiples of the same number.<br>$^{2}/_{3} + {}^{1}/_{6} = {}^{4}/_{6} + {}^{1}/_{6} = {}^{5}/_{6}$ |

| | | | |
|---|---|---|---|
| Stage 2 | Please note: Concrete apparatus and pictorial representations should still be used to support children who may be struggling with abstract concepts. Concrete apparatus should be readily available for lower achieving children and these with SEND. | Children will begin to use the bar model when problem solving. Jottings and calculations should be recorded to show their processes.<br>  <br> | **Column method**<br>Children to further develop their confidence using the column method. Larger numbers, decimal places and inserting zero for place holders when decimals are different.<br><br>Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions<br><br>$^{1}/_{3} + {}^{1}/_{5} = {}^{5}/_{15} + {}^{3}/_{15} = {}^{8}/_{15}$<br><br>Insert zeros for place holders.<br>  <br><br><br>6 digit + 6 digit<br>  <br><br><br>Numbers with 3 decimal place<br>  <br><br><br>Numbers with a different number of decimal places<br><br>45.25 + 8.5 + 3.247<br>  <br> |

# SUBTRACTION - KS1 (Years 1&2)

## Year 1

**-Read, write and interpret mathematical statements involving addition (-) and (=) signs.**

**-Represent and use number bonds and related subtraction facts within 20**

**-Subtract one-digit and two-digit numbers to 20, including zero**

**-Solve one-step problems that involve subtraction using concrete objects and pictorial representations, and missing number problems such as 8 - □ = 5**

**Resources**

Songs and rhymes

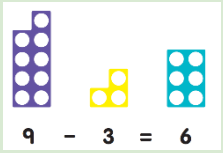
Working with apparatus

Bead strings to 20.

Cubes, dienes, bar model.

Part whole models

Subtraction with Numicon.




Physical and practical work on structured number lines eg jumping backwards

Number stories, 15 people on a bus 3 get off, how many are left on?

Putting a number in your head and counting back with fingers to help.

**Vocabulary**

Counting back

Take away

subtract

## Year 2

**-Solve subtraction problems using concrete objects and pictorial representations, including:**

- **a two-digit number and ones**
- **a two-digit number and tens**
- **two two-digit numbers**

**-Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100**

**-Show that subtraction of one number from another cannot be done in any order.**

**-Use the inverse relationship between addition and subtraction to check calculations and solve missing number problems.**

**Resources**

Number lines and empty number lines

Dienes

Numicon

Part whole models

Place value counters

**Vocabulary**

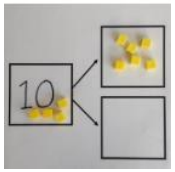
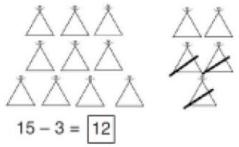
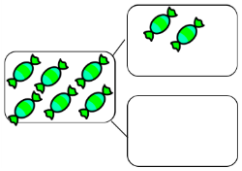
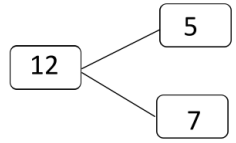
Counting back

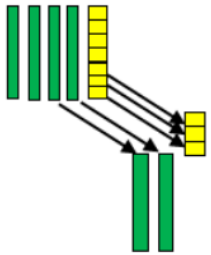
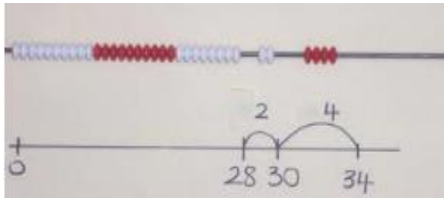
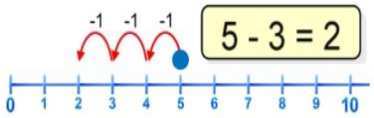
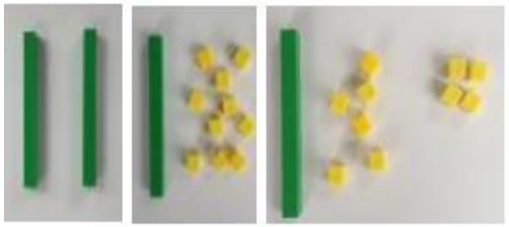
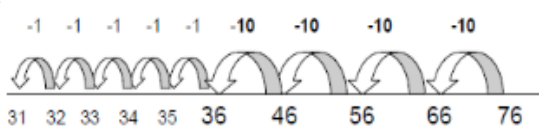
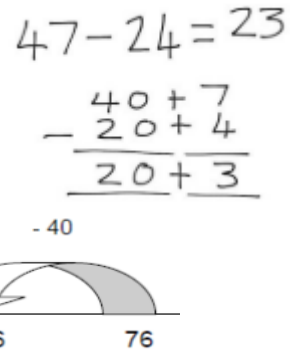
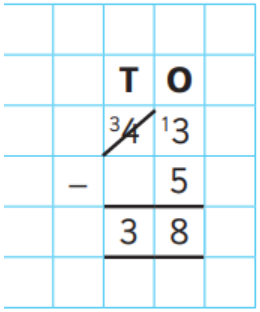
Take away

Subtract

Tens and ones

Find the difference

| | Concrete | Pictorial | Abstract |
|---|---|---|---|
| Stage 1 | **Taking objects away**<br>Use part whole model, cubes and bead strings to subtract two numbers together by moving objects away from the group.<br>  <br>  <br>  <br>  | Use jottings to represent numbers. Children will learn to cross out drawn objects to show what has been taken away.<br>  <br>  | Children will record their calculation using a pictorial method along with a calculation using numbers and symbols.<br>12 - 5 = 7<br>  <br>They may use their mental<br>  <br>their fingers to support methods |

| | | | |
|---|---|---|---|
| Stage 2 | Children will use dienes cubes to subtract larger numbers where exchanging is not required. Children will lay out the first number using the dienes cubes and then move the second number away to show the subtraction.<br>  <br>They will also use a bead string to add larger numbers by counting in tens and ones.<br>  | **Number line**<br>Children will begin to draw their own number lines. Start at the larger number on the number line and count back in ones or in one jump to find the answer.<br>  <br>Numbers will get progressively larger throughout the keystage. Children will be able to subtract tens and ones using an empty number line.<br>Children will show their representations from the concrete method using pictures.<br>  <br>43—21 = 22 | Children will record their calculation using a pictorial method along with a calculation using numbers and symbols.<br><br>25 - 12 = 13<br><br>Children will begin to subtract multiples of tens.<br><br>25 - 10<br>25 - 10 = 15 |
| Stage 3 | Children will begin to use place value counters and dienes cubes to show how to exchange between units of number. They will be able to change 1 ten and exchange it for 10 ones.<br>  | **Empty number line -**Use an empty number line to count back in tens and then ones.<br>  <br>When confident:<br>  | **Partitioning method**<br>Children will begin to use the partitioning method. Tens and ones will be subtracted to form partial sums and then these partial sums will be added together to find the total.<br><br>47 – 23 = 24<br>47 – 20 = 27<br>27 – 3 = 24<br><br>Move on to when understanding of exchanging is secure.<br>  |

# SUBTRACTION - Lower KS2 (Years 3 & 4)

## Year 3

**-Subtract a range of numbers mentally, including:**

- **a three-digit number and ones**
- **three-digit number and tens**
- **a three-digit number and hundreds**

**-Subtract numbers with up to three digits, using formal written methods of columnar subtraction**

**-Estimate the answer to a calculation and use inverse operations to check answers**

**-Solve problems, including missing number problems, using number facts, place value, and more complex addition.**

**-Subtract fractions with the same denominator within one whole (for example, $^5/_7 - ^1/_7 = ^4/_7$)**

**Resources**
Place value counters
Dienes
Empty number lines

**Vocabulary**
Exchange 1 hundred for 10 tens to subtract

## Year 4

**-Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate**

**-Estimate and use inverse operations to check answers to a calculation**

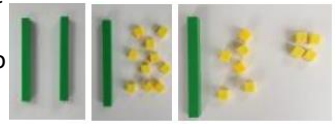
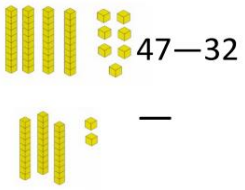
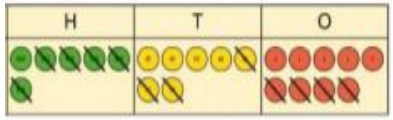
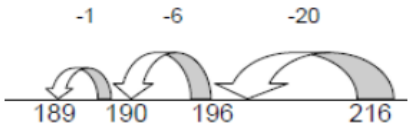
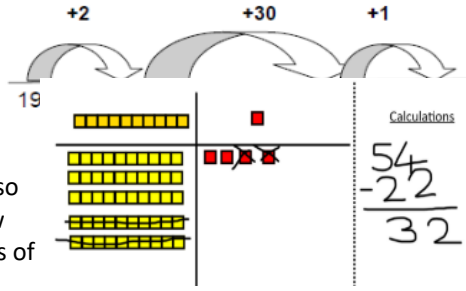
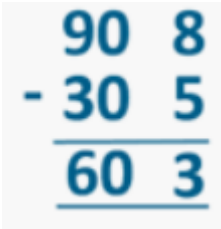
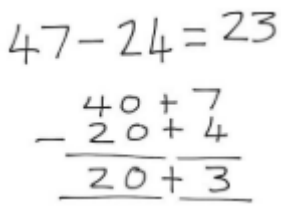
**-Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.**

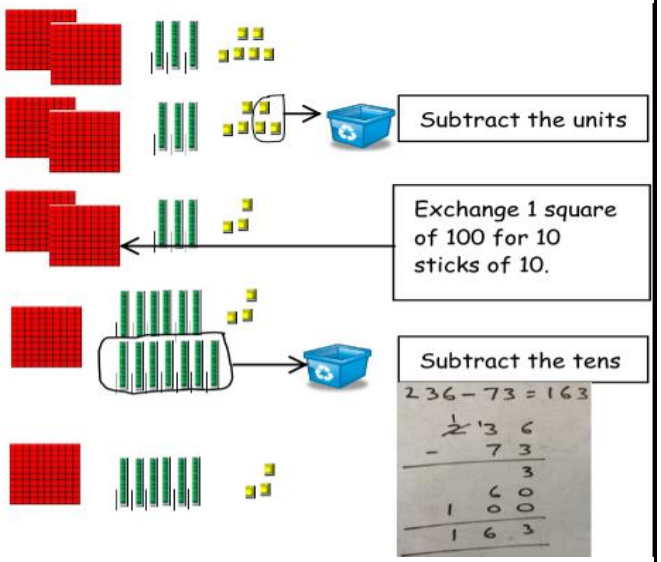
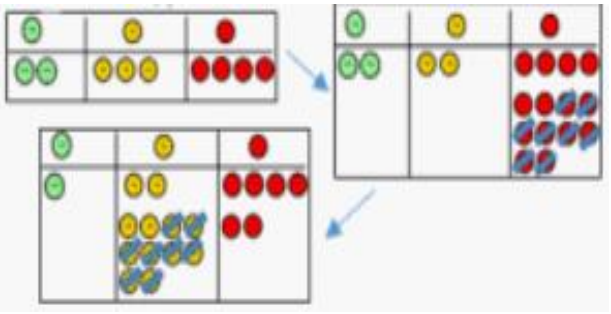
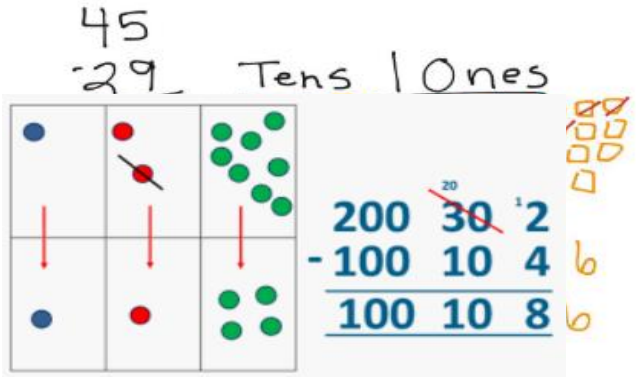
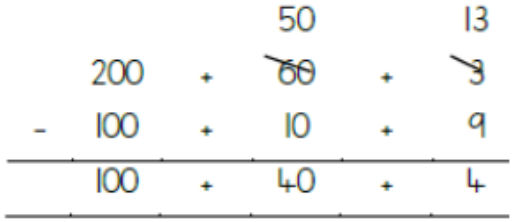
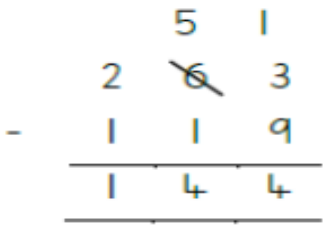
**-Subtract fractions with the same denominator**

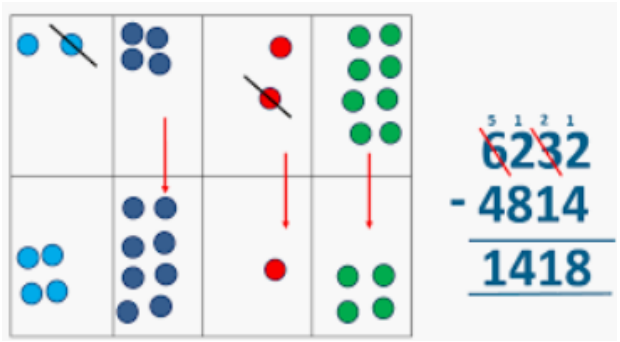
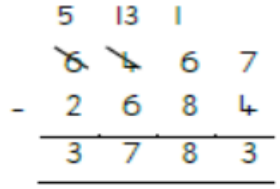
**-Solve simple measure and money problems involving fractions and decimals to two decimal places**

**Resources**
Place value counters
Dienes
Empty number lines

**Vocabulary**
Exchange 1 hundred for 10 tens to subtract

| | Concrete | Pictorial | Abstract |
|---|---|---|---|
| Stage 1 | Children consolidate and use place value counters and dienes cubes to show how to exchange between units of number. They will be able to change 1 ten and exchange it for 10 ones.<br>  <br>They will be able to begin to lay this out like the column method and removing counters or cubes away to represent taking away.<br>  <br>47—32<br>—<br>  <br> | Consolidate their learning from KS1 by using an empty number line to calculate larger numbers.<br>Develop the use of empty number line with calculations that bridge 100:<br>  <br><br>Count on to find small differences:<br>  <br><br>Children will also be able to draw representations of |  <br><br>  <br><br>Children to further secure their knowledge using the **partitioning method** but will lay their work out using the column method approach. Tens and ones will be subtracted to form partial sums and then these partial sums will be added together to find the total. |

| | | | |
|---|---|---|---|
| | | dienes cubes and place value counters by crossing out the number being taken away. | |
| Stage 2 | Children begin to set out HT0 – HT0 using dienes cubes and place value counters (that cross the tens boundary) in columns and record as column subtraction with decomposition (breaking numbers into parts). Children are taught how to exchange units of numbers rather than "borrowing" from the next column. Children will be taught to exchange 1 hundred for 10 tens to make 13 tens, then 13 tens-7 tens- 6 tens. As shown below.<br>  <br>  | Children may draw dienes cubes or place value counters and cross off showing their understanding of taking away. They will need to represent any exchanging that takes place.<br>  | **Partitioning method - with exchanging**<br>Children will use the partitioning method to show exchanging.<br>  <br>Once confident, children can start to use the column method. By this stage children should have a secure understanding of exchanging and the values of each digit.<br>  |

| | | | |
|---|---|---|---|
| Stage 3 | Children continue to develop their confidence in using dienes cubes and place value counters to show decomposition (breaking numbers into parts) using the column method.<br>  | Children draw representations from concrete activities using dienes cubes and place value counters.<br>  | **Column Method**<br>Children continue to use column method to subtract larger numbers.<br>  |

## SUBTRACTION - Upper KS2 (Years 5 & 6)

### Year 5

**-Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)**
**-Subtract numbers mentally with increasingly large numbers (eg. 10 462 - 2300 = 8 162).**
**-Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy**
**-Solve addition and subtraction multi-step problems in contexts, including to 3 decimal places, deciding which operations and methods to use and why.**
**-Add and subtract fractions with the same denominator and denominators that are multiples of the same number**

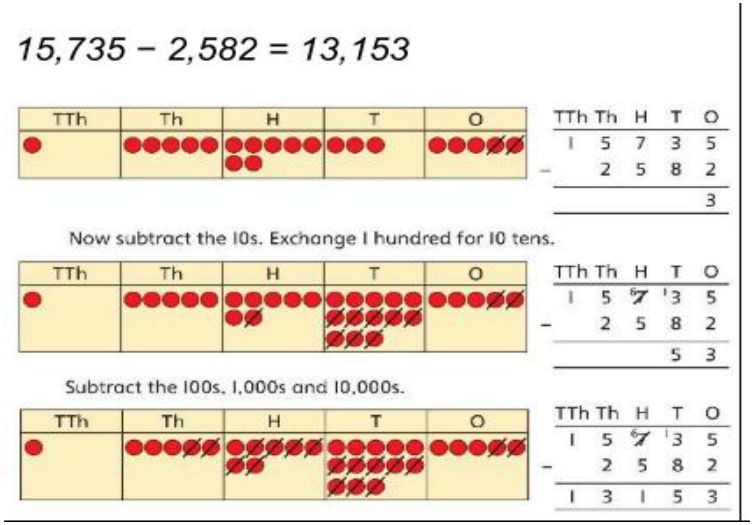
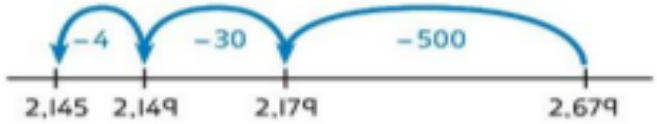
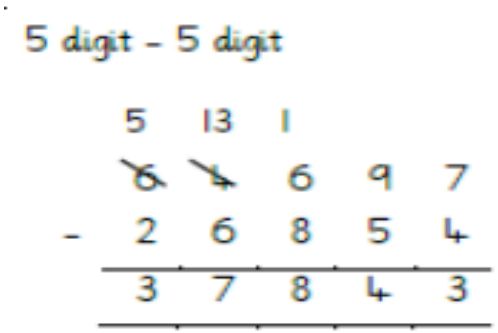
**Resources**
Empty number lines
Decimal place value counters

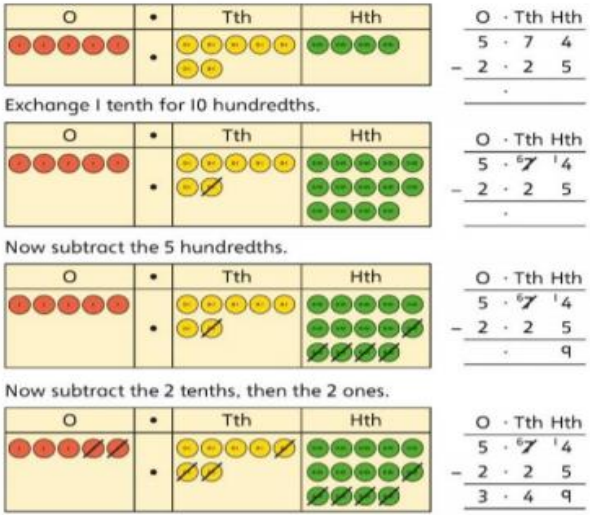
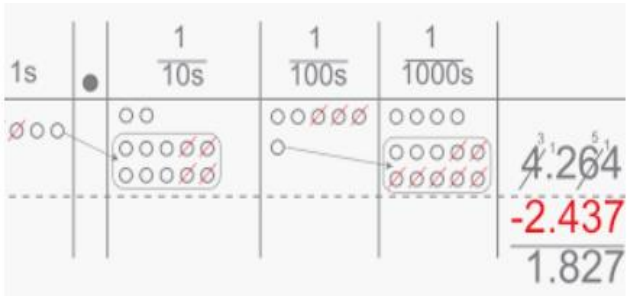
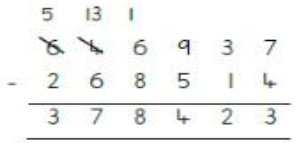
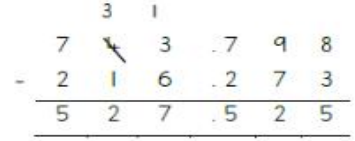
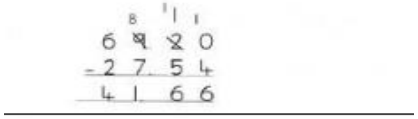
### Year 6

**-Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)**
**-Perform mental calculations, including with mixed operations and large numbers**
**-Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why**
**-Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions**

**Resources**
Empty number lines
Decimal place value counters

| | Concrete | Pictorial | Abstract |
|---|---|---|---|

| | | | |
|---|---|---|---|
| Stage 1 | Please note: Concrete apparatus and pictorial representations should still be used to support children who may be struggling with abstract concepts. Concrete apparatus should be readily available for lower achieving children and these with SEND. | Children can draw using place value counters showing how exchanging takes place between the units of numbers.<br>  <br>Children also show subtraction on an empty number line using larger numbers.<br>  | **Column Method**<br>Children will continue to develop their understanding of column method subtraction. Calculations will become larger.<br>  <br>Subtract fractions with the same denominator and multiples of the same number.<br>$^{2}/_{3} - {}^{1}/_{6} = {}^{4}/_{6} - {}^{1}/_{6} = {}^{3}/_{6}$ |

| | | | |
|---|---|---|---|
| Stage 2 | Introduce decimal place value counters and model exchange for subtracting between units of numbers.<br><br>$5{\cdot}74 - 2{\cdot}25 = ?$<br><br>  <br> | Children will draw their representations showing where they have exchanged.<br><br>  <br> | Children will continue to develop their understanding of column method subtraction. Calculations will become larger, include decimal places and require 0 to be added as a placeholder. 6 digit - 6 digit<br><br>  <br><br><br>Numbers with 3 decimal place<br><br>  <br><br><br>Numbers with a different number of decimal places<br><br>69.2 - 27.54<br><br>  <br><br><br>Subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions<br><br>$^{1}/_{3} - {}^{1}/_{5} = {}^{5}/_{15} - {}^{3}/_{15} = {}^{2}/_{15}$ |

# MULTIPLICATION - KS1 (Years 1&2)

## Year 1

**-Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.**
**-Make connections between arrays, number patterns, and counting in twos, fives and tens**

**Resources**
Bar model
Number shapes
Counters
Bead strings
Number lines

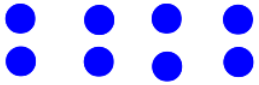
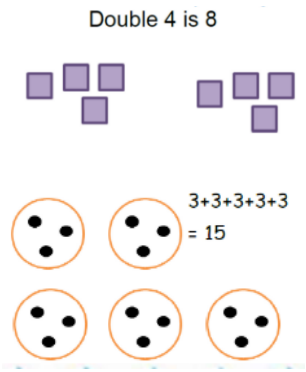
**Vocabulary**
Double
Repeated addition
Multiples
Arrays

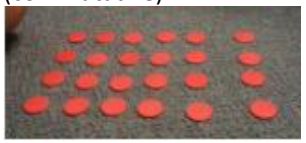
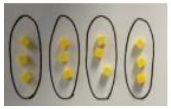
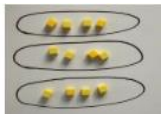
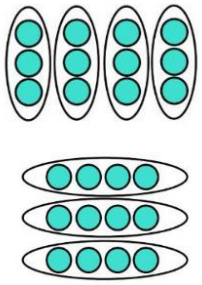
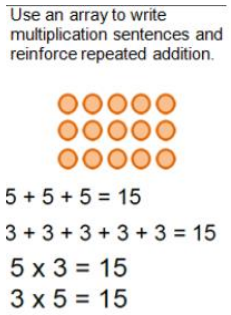
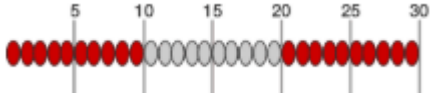
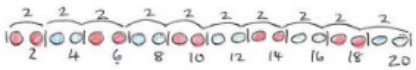
## Year 2

**-Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers**
**-Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs**
**-Show that multiplication of two numbers can be done in any order (commutative)**
**-Solve problems involving multiplication using materials, arrays, repeated addition, mental methods and multiplication and including problems in contexts.**

**Resources**
Bar model
Number shapes
Counters
Bead strings
Number lines

**Vocabulary**
Double
Repeated addition
Multiples
Arrays

| | Concrete | Pictorial | Abstract |
|---|---|---|---|
| Stage 1 | **Repeated addition - Groups of multiple objects**<br>Children will count groups of the same number of objects and add them together. The children learn about grouping in practical contexts and through pictorial representations.<br>  | Children draw representations to show counting in multiples and groups.<br>  <br>  | Children show multiplication as repeated addition. Children may provide pictorial representations to support.<br><br>3 x 9<br>3 + 3 + 3 = 9 |

| | | | |
|---|---|---|---|
| Stage 2 | **Arrays**<br>Children will be able to represent a multiplication calculation using an array and write the multiplication symbol within a number sentence. Children will also understand that multiplication can be carried out in any order (commutative).<br>  <br>3 x 5 = 15<br>5 x 3 = 15<br>    | Children draw representations to show arrays.<br>  | Children use arrays to show how to solve multiplication calculations. Children are able to show that multiplication can be done in any order (commutative).<br>3 x 5 = 15<br>5 x 3 = 15<br>Introduce x sign and record as number sentence<br>7 x 10 = 70<br>4 x 5 = 20<br>  |
| Stage 3 | **Number line**<br>Children will understand the operation of multiplication as repeated addition on a blank number line and will use practical resources to support this. Count the groups as children are skip counting, children may use their fingers as they are skip counting.<br>  <br>  | Children will be able to use an empty number line to show multiplication as repeated addition. The use of beadsting concrete resources may be used to support conceptual understanding.<br>  <br>  <br>  | Children show multiplication as repeated addition.<br>5 + 5 + 5 = 15<br><br>Introduce x sign and record as number sentence<br>7 x 10 = 70<br>4 x 5 = 20 |

# MULTIPLICATION - Lower KS2 (Years 3 & 4)

## Year 3

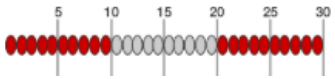
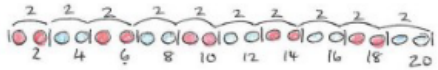
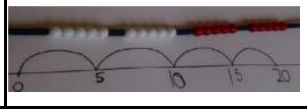
**-Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables**
**-Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods of short multiplication.**
**-Solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.**

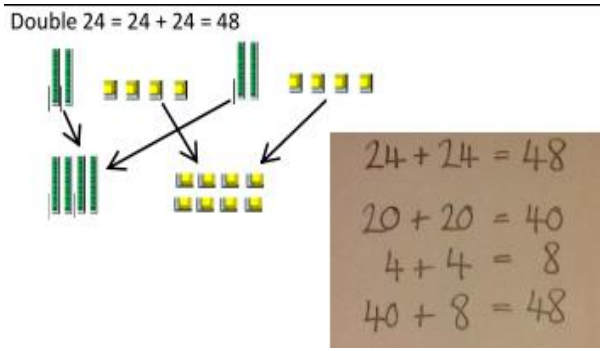
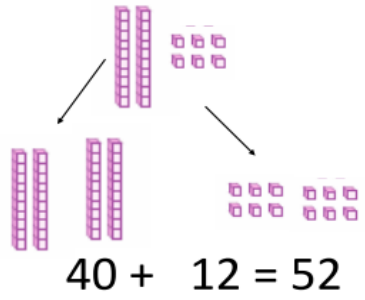
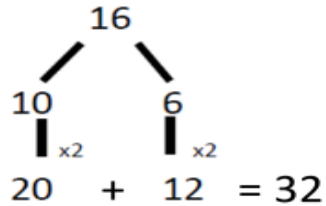
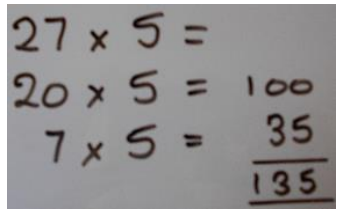
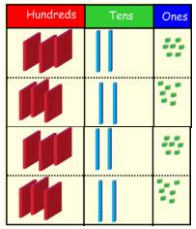
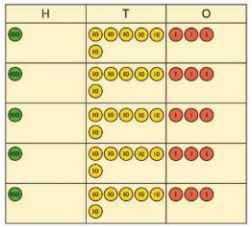
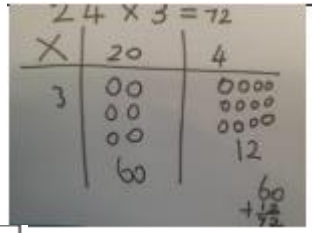
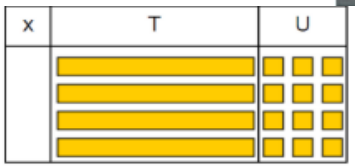
**Resources**
Place value counters
Base 10
Expanded method

## Year 4

**-Recall multiplication and division facts for multiplication tables up to 12 × 12**
**-Use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1 and multiplying together three numbers**
**-Recognise and use factor pairs and commutativity in mental calculations**
**-Multiply two-digit and three-digit numbers by a one-digit number using formal written layout**
**-Solve problems involving multiplying including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.**

**Resources**
Place value counters
Base 10
Grid method
Expanded method
Short written method

| | Concrete | Pictorial | Abstract |
|---|---|---|---|
| Stage 1 | **Number line - Consolidation**<br>Children will understand the operation of multiplication as repeated addition on a blank number line and will use practical resources to support this. Count the groups as children are skip counting, children may use their fingers as they are skip counting.<br>  <br>  <br> | Children will be able to use an empty number line to show multiplication as repeated addition. The use of beadsting concrete resources may be used to support conceptual understanding.<br>  <br><br>  <br><br>  <br> | Children show multiplication as repeated addition.<br>5 + 5 + 5 = 15<br><br>Introduce x sign and record as number sentence<br>7 x 10 = 70<br>4 x 5 = 20 |

| | | | |
|---|---|---|---|
| Stage 2 | **Partitioning**<br>  <br>Children will learn to multiply ones and tens separately before recombing the numbers back together. They can use Dienes cube of place value counters to achieve this. | Children can draw representations of the partitioning process to support their conceptual understanding.<br>  | Partition a number and then multiply each part before recombining it back together.<br>  <br>  |
| Stage 3 | **Grid Method**<br>Show the links with arrays to first introduce the grid method.<br><br>Move onto Dienes cubes to move towards a more compact method.<br><br>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 5 so we need 5 rows of that number.<br>  <br>  | Pictorial representations can be made using their concrete manipulatives as visuals. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown.<br>  <br>  <br>13 x 4 = (10 x 4) + (3 x 4)<br>= 40 + 12<br>= 52 | Children should be able to draw the grid method for each multiplication. The grid method will be used to show how this relates to a formal written method.<br><br>Grid method may then lead to the expanded method.<br>Short written method by ye 4 if confident.<br><br>Grid method<br>X \| 30 \| 6<br>4 \| 120 \| 24<br><br>36 x 4<br>24 (6 x 4)<br>120 (30 x 4)<br>144 |

## MULTIPLICATION - Upper KS2 (Years 5 & 6)

### Year 5

**-Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers**
**-Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers**
**-Multiply numbers mentally drawing upon known facts, including multiplying whole numbers and those involving decimals by 10, 100 and 1000**
**-Recognise and use square numbers and cube numbers, write the notation for both [($^2$) and ($^3$)] and solve problems involving multiplication using knowledge of factors and multiples, squares and cubes**
**-Solve problems involving scaling by simple fractions.**
**-Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams**

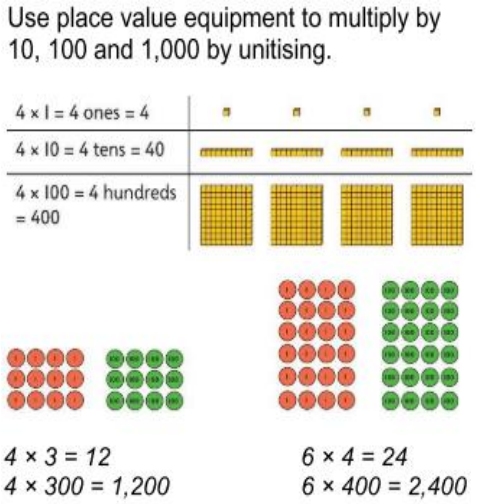
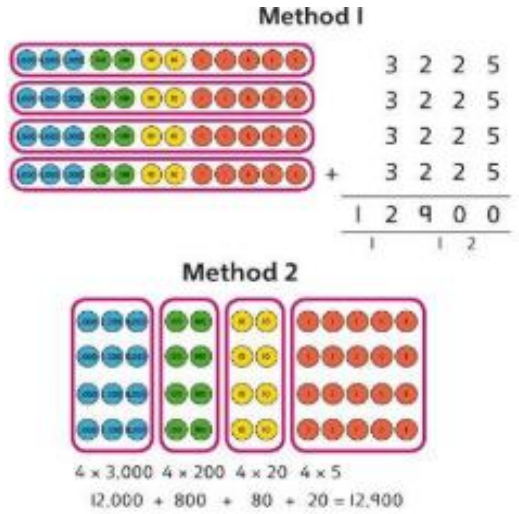
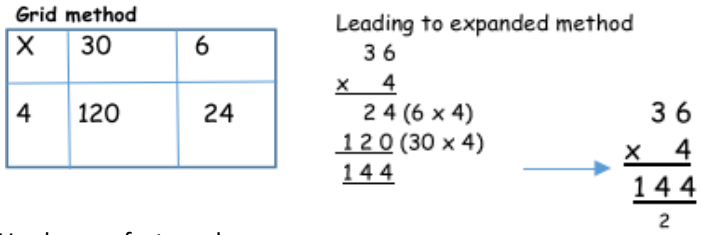
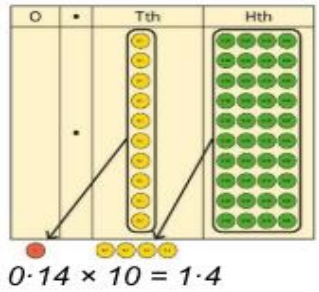
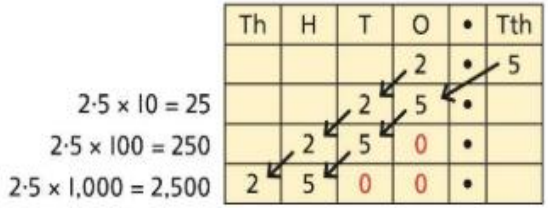
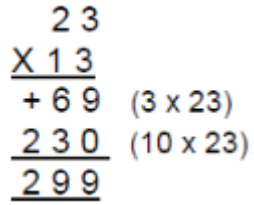
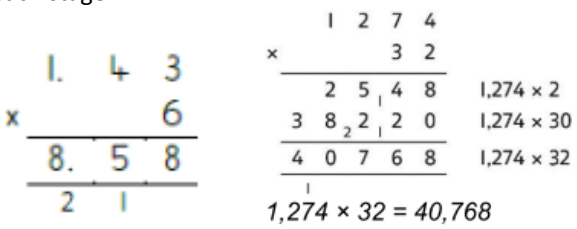
**Resources**
Dienes
Base 10
Place value counters
Grid method
Short written method when multiplying by 1

### Year 6

**Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication**
**-Multiply one-digit numbers with up to two decimal places by whole numbers**
**-Perform mental calculations, including with mixed operations and large numbers**
**-Multiply simple pairs of proper fractions**
**-Identify common factors, common multiples and prime numbers**

**Resources**
Formal written method

| | Concrete | Pictorial | Abstract |
|---|---|---|---|

| | | | |
|---|---|---|---|
| Stage 1 | Concrete materials may be needed to support children's conceptual understanding. Dienes cubes and place value counters will support.<br><br>When multiplying by 10,100,1000 initial concrete resources will be used to support understanding.<br><br>  | Use place value equipment to compare methods.<br><br>  | The **grid method** may be used to show how this relates to a formal written method.<br>Grid method will lead onto **expanded method** and then onto the **compact short multiplication.**<br><br>  <br><br>Use known facts and unitising to multiply.<br>5 × 4 = 20<br>5 × 40 = 200<br>5 × 400 = 2,000<br>5 × 4,000 – 20,000<br>5,000 × 4 = 20,000 |
| Stage 2 | When multiplying decimals by 10,100,1000 initial concrete resources will be used to support understanding to show how exchanging can take place.<br><br>  | This pictorial grid method will support children's understanding of multiplying by 10, 100, 1000.<br><br>  | **Long multiplication**<br>Children may wish to use 2 separate calculations to support their understanding. Reinforce language of place value when multiplying by multiples of 10. Extend to 3 or 4-digit numbers multiplied by a 2-digit number.<br><br>  |
| Stage 3 | Please note: Concrete apparatus and pictorial representations should still be used to support children who may be struggling with abstract concepts. Concrete apparatus should be readily available for lower achieving children and these with SEND. | Please note: Concrete apparatus and pictorial representations should still be used to support children who may be struggling with abstract concepts. Concrete apparatus should be readily available for lower achieving children and these with SEND. | Use column multiplication, ensuring understanding of place value at each stage.<br><br>  |

# DIVISION - KS1 (Years 1&2)

## Year 1

**-Solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.**

**Resources**
Practical activities, songs, and games, cubes

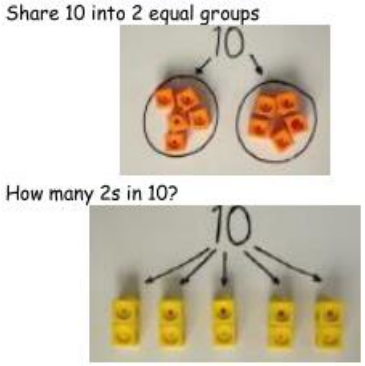
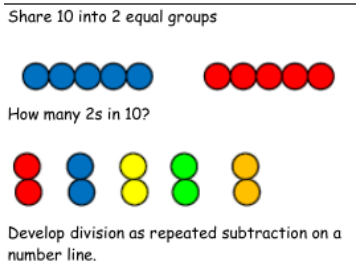
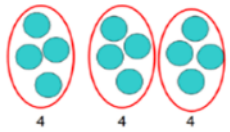
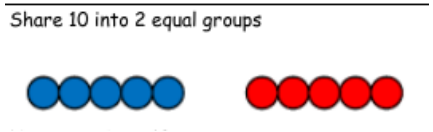
**Vocabulary**
Sharing, grouping, equal, divide, division, half

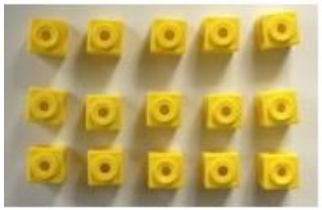
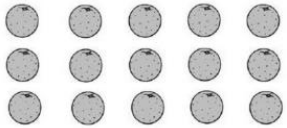
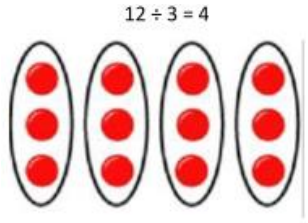
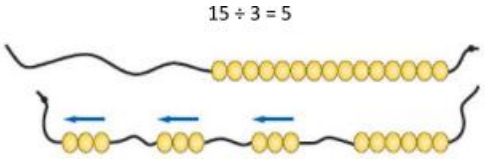
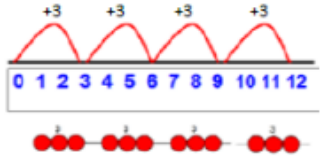
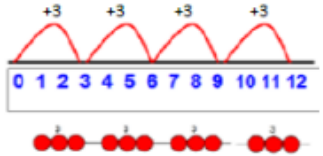
## Year 2

**-Recall and use division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers**
**-Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs**
**-Show that division of one number by another cannot be done in any order**
**-Solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and multiplication and division facts, including problems in contexts.**

**Resources**
Practical activities, songs, and games, cubes

**Vocabulary**
Sharing, grouping, equal, divide, division

| | Concrete | Pictorial | Abstract |
|---|---|---|---|
| Stage 1 | **Sharing and Grouping**<br>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.<br>  | Use pictures to share objects. Use circles rather than dots to aid counting.<br>  <br>Develop division as repeated subtraction on a number line.<br>  <br>12 shared between 3 is 4 | Children will be able to represent a division calculation using a pictorial method and write the division within a number sentence.<br>10 ÷ 2 = 5<br>  |

| | | | |
|---|---|---|---|
| Stage 2 | **Arrays**<br>Link division to multiplication by creating an array and thinking about the number sentences that can be created.<br><br>Eg:<br>15 ÷ 3 = 5 5 x 3 = 15<br>15 ÷ 5 = 3 3 x 5 = 15<br><br>  | Draw arrays to show how pictures are divided.<br><br>  | Children will be able to represent a division calculation using an array and write the division within a number sentence<br><br>  |
| Stage 3 | **Repeated addition and subtraction**<br>Children will understand the operation and repeated addition or subtraction using bead strings and number lines. This will support the pictorial element.<br><br>  | Children will understand the operation of division as grouping using repeated addition or subtraction on a prepared number line.<br><br>  <br>**12 ÷ 3 = 4** | Children will be able to represent a division calculation using a numberline and write the division within a number sentence.<br><br>  <br>**12 ÷ 3 = 4** |

## DIVISION - Lower KS2 (Years 3 & 4)

**Year 3**

**-Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables**
**-Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, using mental and progressing to formal written methods**
**-Solve problems, including missing number problems, involving multiplication and division**
**-Calculate simple remainders after division**

**Resources**
Base ten, place value counters, blank number lines

**Vocabulary**
Groups of, exchange

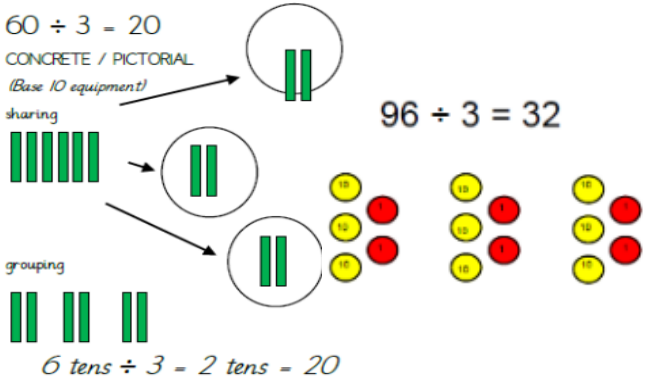
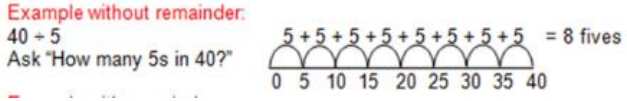
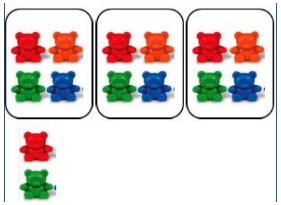
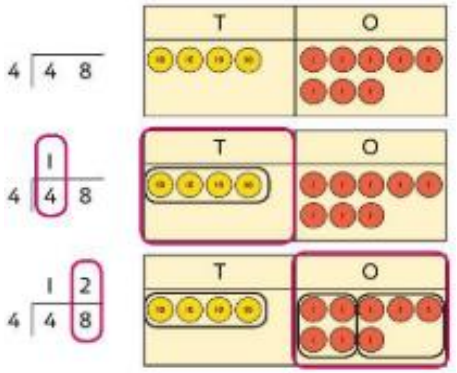
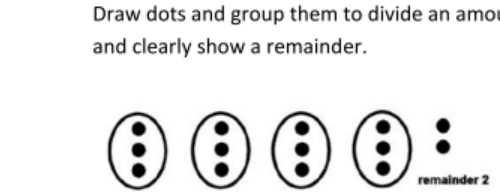
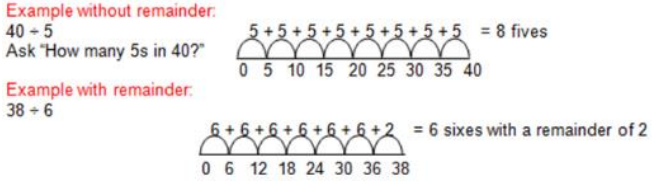
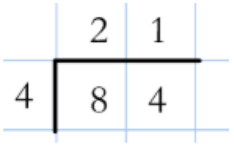
**Year 4**

**-Recall multiplication and division facts for multiplication tables up to 12 × 12**
**-Recognise and use factor pairs in mental calculations**
**-Divide two-digit and three-digit numbers by a one-digit number using formal written layout**
**-Divide a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths**

**Resources**
Base ten, place value counters, blank number lines

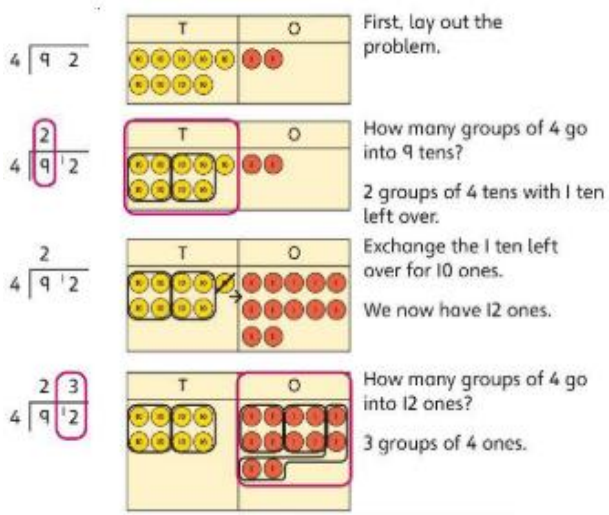
**Vocabulary**
Groups of, exchange

| | Concrete | Pictorial | Abstract |
|---|---|---|---|

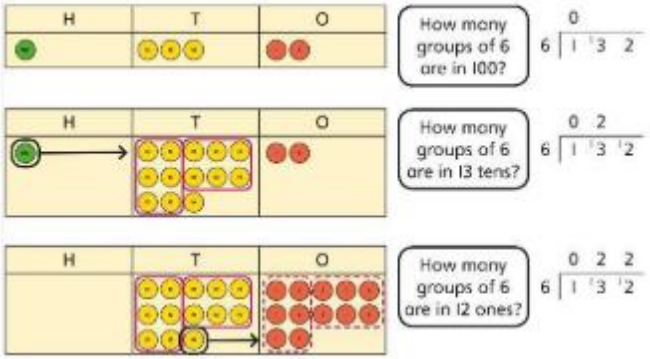
| | | | |
|---|---|---|---|
| Stage 1 | **Division with no remainders through sharing**.<br>Use concrete materials to share into groups.<br>  | Consolidate learning from KS1 using diagrams of sharing and repeated subtraction and addition on a number line to make jumps<br>  <br>Concrete methods could be represented pictorially within books to show understanding. | How many groups of 6 in 24?<br><br>24 ÷ 6 = 4<br><br>Abstract methods may be supported with pictorial methods within the children's books. |
| Stage 2 | **Division with remainder through sharing**<br>14 ÷ 3 =<br>Divide objects between groups and see how much is left over.<br>  <br>**Division no remainders - introduction to bus stop method**<br>Use place value equipment on a place value grid alongside short division. The model uses grouping.<br>  | Students can continue to use drawn diagrams with circles to help them divide numbers into equal groups.<br>Remainders will be seen by not fitting into a whole group.<br>  <br>  | Children will begin to move onto division with remainders. A number sentence will support any abstract written calculation by using pictorial method to support.<br>  <br>**Short division**<br>Children will begin to use the formal written method of division without remainders. This will only come after a clear concept is understood using manipulatives.<br>  <br>Dividing by 2,3,4, and 5 |

Stage 3

**Division with remainders - Bus stop**

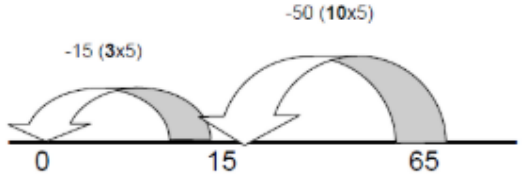



Pictorial representations can be used to support any concrete manipulatives.




Continue to use blank number lines as appropriate, using multiples of the divisor.

65 ÷ 5 = 13




**Bus Stop Method for division**

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 2\ 1\ 8 \\ 4\overline{)\,8\ 7\ 2} \\ \quad\ 3 \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 8\ 6\ r\ 2 \\ 5\overline{)\,4\ 3\ 2} \\ \quad\ 3 \end{array}$$

**DIVISION - Upper KS2 (Years 5 & 6)**

**Year 5**

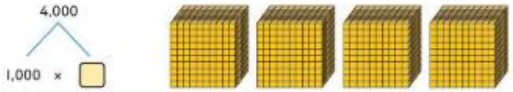
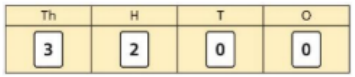
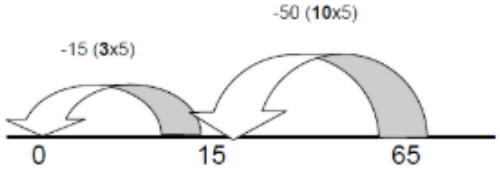
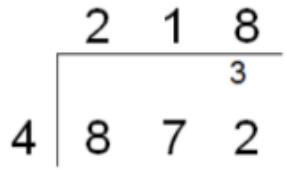
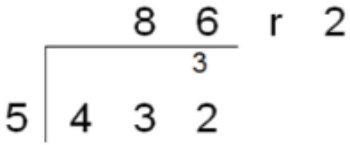
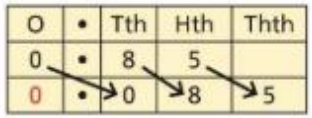
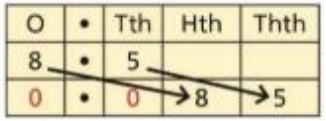
**Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context**
**-Divide whole numbers and those involving decimals by 10, 100 and 1000**
**-Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers**
**-Establish whether a number up to 100 is prime and recall prime numbers up to 19**

**Resources**
Base 10
Blank number lines

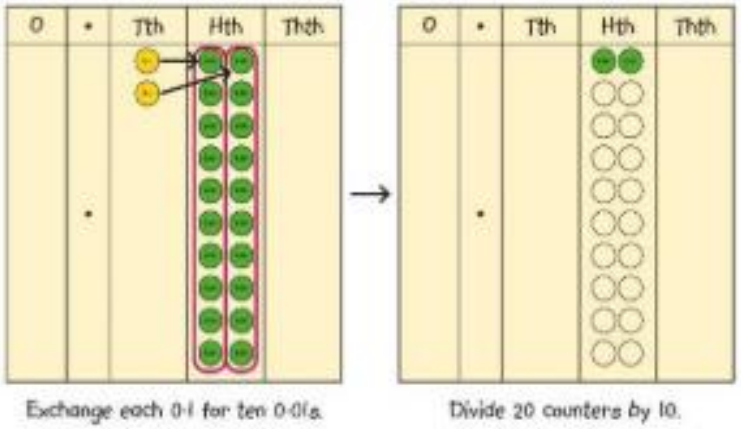
**Year 6**

**-Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context**
**-Identify common factors, common multiples and prime numbers**
**-Divide proper fractions by whole numbers (for example, $^1/_3 \div 2 = {}^1/_6$)**
**-Associate a fraction with division and calculate decimal fraction equivalents (for example, 0.375) for a simple fraction (for example, $^3/_8$)**

| | Concrete | Pictorial | Abstract |
|---|---|---|---|

| | | | |
|---|---|---|---|
| Stage 1 | **Dividing whole numbers by 10, 100 and 1,000**<br>Use place value equipment to support unitising for division.<br>*4,000 ÷ 1,000*<br>  <br>*4,000 is 4 thousands.*<br>*4 × 1,000= 4,000*<br>*So, 4,000 ÷ 1,000 = 4*<br>Concrete and pictorial representations may still be required to support the formal method of division (Bus Stop) - Go back to LKS2 to consolidate learning. | Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000.<br>  <br>*3,200 ÷ 100 = ?*<br>*3,200 is 3 thousands and 2 hundreds.*<br>*200 ÷ 100 = 2*<br>*3,000 ÷ 100 = 30*<br>*3,200 ÷ 100 = 32*<br>*So, the digits will move two places to the right.*<br>Continue to use blank number lines as appropriate, using multiples of the divisor.<br>65 ÷ 5 = 13<br>  | **Bus Stop Method for division**<br>Begin with divisions that divide equally with no remainder.<br>  <br>Move onto divisions with a remainder.<br>  |
| Stage 2 | **Dividing decimals by 10, 100 and 1,000**<br>Use place value counters to represent dividing by 10, 100, 1000. Represent division using exchange on a place value grid.<br>0·2 is 2 tenths.<br>2 tenths is equivalent to 20 hundredths. | Represent division to show the relationship with multiplication. Understand the effect of dividing by 10, 100 and 1,000 on the digits on a place value grid.<br>  <br>*0·85 ÷ 10 = 0·085*<br>  <br>*8·5 ÷ 100 = 0·085* | |

20 hundredths divided by 10 is 2 hundredths.




Stage 3

**Long Division - Abstract Method**

Calculations will start with tens and ones and move onto more advanced division calculations.

Find how many groups of 19 tens are in 58 tens. 3 groups equal 57 tens. Write 3 above and 57 under 58. Subtract 57 from 58 to leave 1 ten and show this by writing under the tens column. Still have 9 ones, so bring down next to ten to make 19. How many groups of 19 ones can be made? 1. Show this by writing above line. Subtract 19 ones to leave 0.

| | | 3 | 1 |
|---|---|---|---|
| 19 | 5 | 8 | 9 |
| – | 5 | 7 | |
| | | 1 | 9 |
| – | | 1 | 9 |
| | | | 0 |

589 ÷ 19

| 19 | 38 | 57 | 76 | 95 | 114 | 133 | 152 | 171 | 190 |
|---|---|---|---|---|---|---|---|---|---|

4,320 ÷ 32

| 32 | 64 | 96 | 128 | 160 | 192 |
|---|---|---|---|---|---|

| | | 1 | 3 | 5 |
|---|---|---|---|---|
| 32 | 4 | 3 | 2 | 0 |
| – | 3 | 2 | | |
| | 1 | 1 | 2 | |
| – | | 9 | 6 | |
| | | 1 | 6 | 0 |